Math 118 Final  Spring, 2012  Name: __________________

All questions 2 points unless otherwise indicated.
1) Given the formula, \( a_n = 3 + 2^n \)
   a) Write the sequence using the three-dot notation, giving the first four terms.

   b) Give the 100\(^{th}\) term of the sequence. (Leave as an expression.)

2) Give the first four terms of the specified recursively defined sequence.
   \( a_1 = 4, a_2 = 7 \) and \( a_{n+2} = a_{n+1} - a_n \) for \( n \geq 1 \)

3) Evaluate the arithmetic series: \( 1,403 + 1,396 + 1,389 + \ldots + 24 + 17 + 10 \)

4) Evaluate the geometric series: \( 4 + 28 + 196 + \ldots + 4 \cdot 7^{100} \). Leave the answer as an expression.

5) Evaluate \( \sum_{k=1}^{\infty} \frac{8}{5^k} \)
6) Express 0.142142142... as a fraction; here the digits 142 keep repeating forever

7) What is the angle between the hour hand and the minute hand on a clock at 4 o'clock?

8) Assume the earth is a sphere with a diameter of 7926 miles. Approximately how far does a ship travel when sailing along the equator in the Atlantic Ocean from longitude 20º west to longitude 30º west?

9) Suppose an ant walks counterclockwise on the unit circle from the point (−1, 0) to the endpoint of the radius corresponding to 6 radians. How far has the ant walked? Round to 3 decimal places)
10) Suppose a wedge-slice from a 10-inch diameter pizza has an area of 15 square inches. What is the angle of this slice?

11) Suppose $\frac{\pi}{2} < \theta < \pi$ and $\sin \theta = \frac{2}{9}$. Evaluate $\cos \theta$. Give an exact answer.

12) Evaluate $\tan \left( \frac{5\pi}{4} \right)$. Give an exact answer.

13) Find the equation of the line in the xy-plane that contains the point (4, 2) and makes an angle of 65° with the positive x-axis.
14) Given that \( \cos \frac{\pi}{12} = \frac{\sqrt{2} + \sqrt{3}}{2} \) and \( \sin \frac{\pi}{8} = \frac{\sqrt{2} - \sqrt{2}}{2} \)

a) Find the exact expression of \( \cos \left( -\frac{\pi}{12} \right) \)

b) Find the exact expression of \( \sin \left( -\frac{\pi}{8} \right) \)

15) Without a calculator evaluate \( \cos^{-1} \left( \frac{\sqrt{3}}{2} \right) = \theta \) for ALL \( \theta \) in \([0, 2\pi]\).
Give the answer in exact radians.

16) Suppose \( \sin u = \frac{1}{3} \). Evaluate \( \cos u \).
17) Using the right triangle, suppose $a = 3$ and $c = 4$. Evaluate $u$ in radians. (Round to 3 decimal places.)

18) Using the right triangle, suppose $b = 3$ and $v = 38^\circ$.

a) Evaluate $u$ in degrees.

b) Evaluate $c$. (Round to 3 decimal places.)

c) Evaluate $a$. (Round to 3 decimal places.)

19) Find the area of a parallelogram that has pairs of sides of lengths 5 and 11, with a $28^\circ$ angle between two of those sides.

20) Suppose $a = 4$, $B = \frac{2\pi}{11}$, $C = \frac{3\pi}{11}$ radians

Evaluate $A$.

Evaluate $b$

Evaluate $c$
21) Consider the parametric curve described by \((2t^2 - 2t + 1, t^3 - 8t^2 + 14t)\) for \(t\) in the interval \([0, 3]\).

a. What is the initial point of the parametric curve?

b. What is the endpoint of the parametric curve?

c. Sketch the parametric curve.

22) Omitted

23) (1 pt each) Consider the function \(f(x) = -\cos x + 4\).

a) What is the midline?

b) What is the amplitude?

c) What is the period?

d) What is the range?
24) (1 pt each) Consider the graph

- What is the midline?
- What is the amplitude?
- What is the period?
- What is B?
- Decide if this is the graph more closely resembles a cosine or a sine function. Write the equation of the function.

25) (1 pt each) A city averages 14 hours of daylight in June, averages 10 in December, and averages 12 in both March and September. Assume that the number of hours of daylight varies sinusoidally over a period of one year.

- What is the midline?
- What is the amplitude?
- What is the period?
- What is B?

**Extra credit:** (2 pts) Write an expression for $n$, the number of hours of daylight, as a cosine function of $t$, that is, $n(t) =$. Let $t$ be in months and $t = 0$ correspond to the month of January. This is difficult. Try your best.
26) Convert the point with the given polar coordinates, \( \left( 9, -\frac{\pi}{3} \right) \) to rectangular coordinates \((x, y)\).

27) Convert the point \((0, 2\pi)\), rectangular coordinates, to polar coordinates \((r, \theta)\). Use radians, rounding to three decimal points, and choose the angle \(\theta\) to be in the interval \((-\pi, \pi]\).

28) Convert the point \((-5, -2)\), rectangular coordinates, to polar coordinates \((r, \theta)\). Use radians, rounding to three decimal points, and choose the angle \(\theta\) to be in the interval \((-\pi, \pi]\).

29) Evaluate \(|7 + 12i|\).

30) Write the complex number, \(2 - 2i\), in polar form.
31). Evaluate \((-3 + 3\sqrt{3}i)^{555}\)

Extra Credit:
A basketball is thrown from height 5 feet with initial horizontal velocity 25 feet per second and initial vertical velocity 13 feet per second. How far away from the thrower (in the horizontal direction) is the basketball when it hits the ground? Round the answer to 2 decimal places.
Extra Credit: Given, $a = 6$, $b = 13$, and $A = 56^\circ$, how many possible triangles can be constructed?

(1 pt) 
A) No triangle  
B) One triangle  
C) Two Triangles  
D) Three or more triangles

(3 pts) Prove your answer: (You know enough geometry from this course to do so.)